Single Slab Arbitrary Polarization Surface Wave Structure*

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Summary—A single grounded dielectric slab can support either TM or TE modes, but cannot propagate both with the same velocity. This paper concerns a modification of the single slab which enables either polarization to propagate with the same velocity. Such a structure could transmit a circularly polarized wave, and would be useful in transmission, feeder, and antenna applications.

The structure consists of a grounded dielectric slab with parallel metal plates imbedded in the dielectric, normal to and in contact with the ground plane. The plates do not reach the top of the slab. Propagation is *along* the plates, whereas corrugated surfaces propagate across the vanes. For small plate thickness, the TE field is undisturbed; hence, the entire slab thickness controls the velocity. The TM field, however, has an electric field component parallel to the plates, which is shorted out by the plates; thus, only the thickness of slab above the plates controls this mode, and the two modes can be independently controlled.

Since the plates are not a perfect short circuit, a boundary value analysis is given which finds the higher mode amplitudes, and the variation of effective short circuit with parameters. This analysis sets up a sum of modes in each region, and then solves the resulting sets of simultaneous transcendental equations by a contour integration-residue theory technique. The theory is illustrated by a specific example.

Introduction

SURFACE WAVE structures have received much attention in the literature during an interval of over fifty years. Most of the interest has been centered on two structures of practical importance: the corrugated metallic surface, and the dielectric surface, with or without an associated ground plane.[1,2] An excellent survey of the state of the art is given by Zucker, with 86 references.[3] Most of the surface wave antennas are of the endfire type.[4] All these structures, however, are essentially single polarization devices. The corrugated surfaces support only TM modes. A dielectric clad ground plane will support either TM or TE modes, but the propagation constants vary with the physical parameters in different fashions. It is not possible to design a single grounded dielectric slab to propagate both polarizations with the same velocity. Hence the array factors for the two polarizations differ when a given slab is used as an antenna.

In some transmission and antenna applications it is desirable to utilize surface wave structures with circular polarization. Two structures which can be designed to propagate both TM and TE surface waves with the same velocity are the two-layered dielectric slab with ground plane and the single-layer grounded slab with mode filter. The latter structure is the subject of this paper.

This structure, called the Single Slab Circular Polarization Structure, consists of a single dielectric layer on a ground plane with parallel metal plates or septa imbedded in the dielectric, normal to and in contact with the ground plane. Fig. 1 is an artist's sketch of the

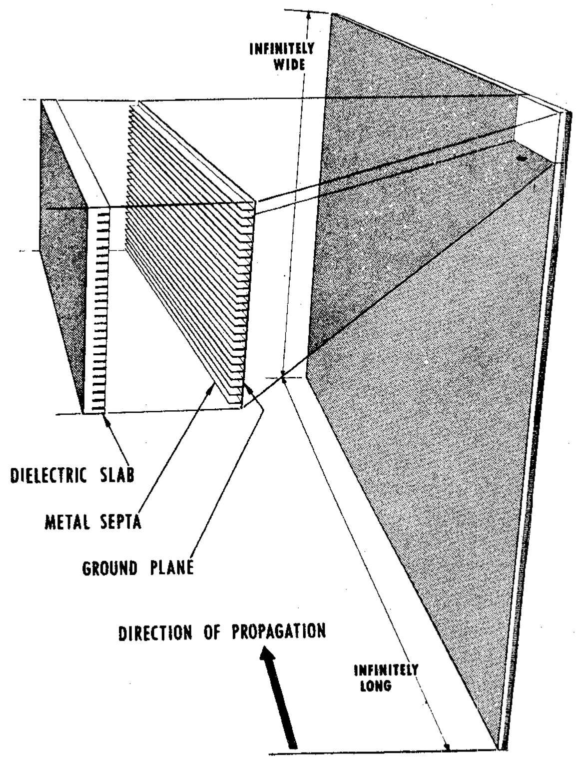

Fig. 1—Single slab arbitrary polarization structure.

structure. Note that propagation is along the vanes while corrugated surfaces propagate a wave across the teeth. The TE mode has an electric field across the vanes, representing the TEM mode in a parallel plate transmission line; for small plate thickness the effect of

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† Hughes Aircraft Co., Culver City, Calif.

[1] S. S. Attwood, "Surface wave propagation over a coated plane conductor," *J. Appl. Phys.*, vol. 22, pp. 504–509; April, 1951.

[2] R. S. Elliott, "On the theory of corrugated plane surfaces," IRE Trans., vol. AP-2, pp. 71–81; April, 1954.

[3] F. J. Zucker, "The guiding and radiation of surface waves," *Proc. of Symp. on Modern Advances in Microwave Techniques*, Polytech. Inst. of Brooklyn, pp. 403–435; 1954.

[4] R. S. Elliott, "Pattern Shaping with Surface Wave Antennas," Tech. Memo. no. 360, Hughes Aircraft Co., March, 1955.

the plates is negligible and the entire slab controls the TE surface wave mode. For the TM mode, however, a parallel component of electric field is present. If the plates are sufficiently close together, this parallel electric field is shorted out, and the effective thickness of slab controlling the TM wave is nearly the thickness above the septa. It is the goal of this paper to determine the quantitative behavior of this effective thickness with the dimensions of the structure.

Single Grounded Slab Modes

The analysis of TM and TE surface waves on a single grounded dielectric (or permeable) slab is straightforward, and may be found in the literature.[5] Only the results are given here. If the surface wave propagation constant is β, and $k=\omega\sqrt{\mu\epsilon}$ for free space then the parameters of interest are the velocity ratio β/k and the slab thickness kc. The formulas are[5]

$$\text{TM} \qquad kc = \frac{\tan^{-1} \epsilon_1 \sqrt{\dfrac{(\beta/k)^2 - 1}{\epsilon_1 - (\beta/k)^2}}}{\sqrt{\epsilon_1 - (\beta/k)^2}} \tag{1}$$

$$\text{TE} \qquad kc = \frac{\sin^{-1} \sqrt{\dfrac{\epsilon_1 - (\beta/k)^2}{\epsilon_1 - 1}}}{\sqrt{\epsilon_1 - (\beta/k)^2}} \cdot \tag{2}$$

which give the thickness required for a slab of dielectric constant ϵ_1, to propagate each mode with velocity ω/β. The maximum value β/k can attain for either mode is $\sqrt{\epsilon_1}$. A typical curve of β/k vs slab thickness is Fig. 2 where it is seen that the TM mode is a dominant mode.

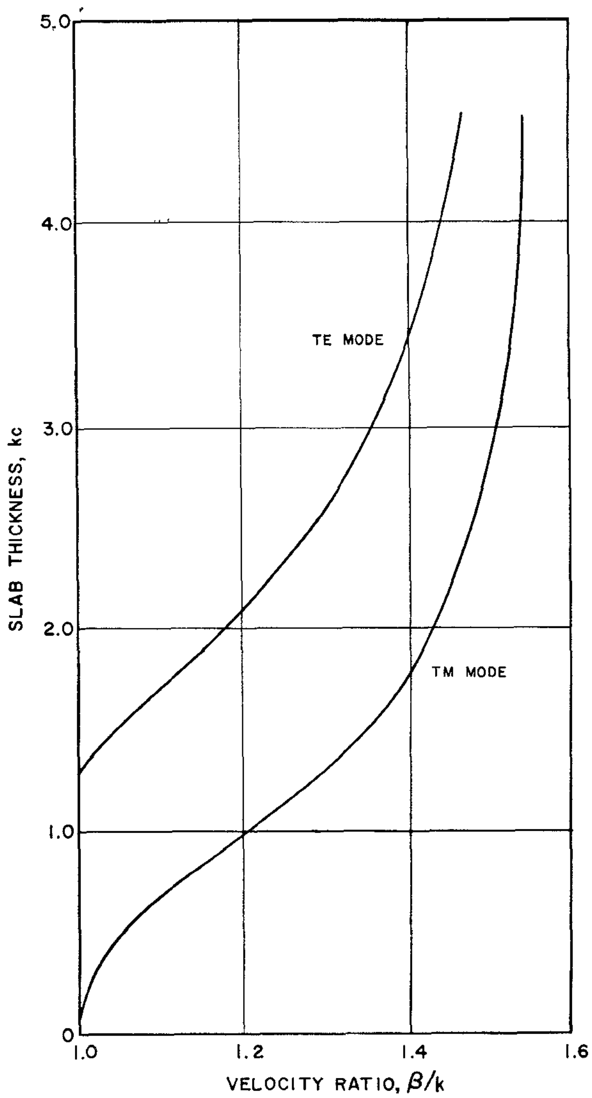

Fig. 2—Velocity ratio for a slab of $\epsilon=2.5$.

The Single Slab Arbitrary Polarization Structure

The single slab structure with septa was sketched in Fig. 1, and is shown in cross section in Fig. 3. The major effect of the septa is on the TM mode; thus this problem will be attacked. Surface waves are again the desired phenomenon; the lowest order TM mode will be assumed to propagate in the z direction as exp $(-j\beta z)$. Unlike the simple case in the section above, a single mode cannot exist either in the dielectric-air region or in the parallel plate waveguide region. Instead, each region has an infinite set of coupled modes, which satisfy boundary conditions and, of course, Maxwell's equations. Waves other than surface waves may exist on the structure, but attention here will be limited to the surface waves. The complete field in each region is written and the tangential fields at the boundaries are matched. This results in the usual infinite set of simultaneous equations in the unknown amplitudes and propagation constants. A residue theory technique is then used to solve the equations. This scheme augments the physical understanding by making explicit approximations but appears to be applicable only for plates of quite small thickness. The steps will be described briefly.

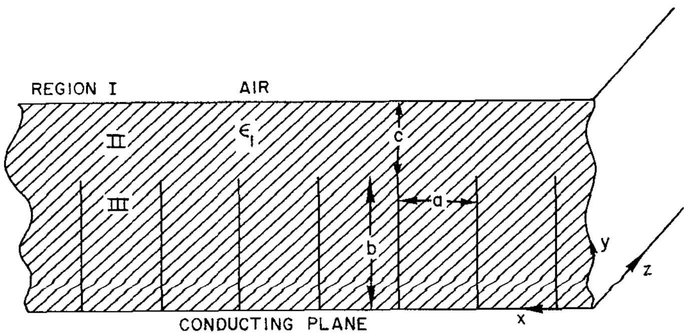

Fig. 3—Single slab structure in transverse section.

[5] Robert C. Hansen, "Single Slab Circular Polarization Surface Wave Structure," Sci. Rep. No. 9, Hughes Aircraft Co., February 14, 1956.

Field Equations in Three Regions

Except for the dominant TM mode, the field is composed of modes, transverse magnetic with respect to $\mathbf{y}$, derived from the scalar wave solution f:

$$\begin{aligned} \boldsymbol{E} &= \frac{1}{j\omega\epsilon} \text{ curl curl } \mathbf{a}_y f, \\ \boldsymbol{H} &= \text{curl } \mathbf{a}_y f. \end{aligned} \tag{3}$$

The reasons for using this type of field will be explained later. A y-directed unit vector is symbolized by $\mathbf{a}_y$. Since the parallel plate region is nonuniform in $\mathbf{x}$, the higher modes will vary in $\mathbf{x}$; from this variation may be deduced the $\mathbf{y}$ variation. Surface waves with exp $(-j\beta \mathbf{z})$ are assumed. This assumption of a single propagation constant for the set of modes is valid because at each boundary the fields must match for all $\mathbf{z}$. The factor exp $(-j\beta z)$ will be suppressed throughout.

In Region II the field must be periodic in $\mathbf{x}$ with period $\mathbf{a}$ due to symmetry. The pythagorean relation forces the higher modes to be evanescent in $\mathbf{y}$. Again it is stressed that propagation is parallel to the plates. Thus,

$$f = B_m \cos \frac{2m\pi x}{a} \sinh (D_m - \gamma_m y) \quad m \geq 1 \tag{4}$$

$$\epsilon_1 k^2 = \beta^2 + \left(\frac{2m\pi}{a}\right)^2 - \gamma_m{}^2. \tag{5}$$

The dominant mode is

$$\begin{aligned} E_y &= -(\beta\gamma/\omega\epsilon\epsilon_1) B \cos (D - \gamma y) \\ E_z &= -(\gamma^2/j\omega\epsilon\epsilon_1) B \sin (D - \gamma y) \\ H_x &= \gamma B \cos (D - \gamma y) \end{aligned} \tag{6}$$

where B is again the dominant mode coefficient. A device which allows this mode to be combined with the higher modes is that of replacing dominant mode coefficients by new symbols subscripted zero. Let

$$jB/\beta = B_0/\gamma_0, \quad j\gamma = \gamma_0, \quad D_0 = j(D + \pi/2). \tag{7}$$

Three of the components, which include the dominant mode, are

$$E_y = \sum_{m=0}^{\infty} \frac{B_m}{jw\epsilon\epsilon_1}\left[\left(\frac{2m\pi}{a}\right)^2 + \beta^2\right] \cos \frac{2m\pi x}{a} \cdot \sinh (D_m - \gamma_m y) \tag{8}$$

$$E_z = \sum_m (\beta\gamma_m B_m / w\epsilon\epsilon_1) \cos \frac{2m\pi x}{a} \cosh (D_m - \gamma_m y) \tag{9}$$

$$H_x = \sum_m j\beta B_m \cos \frac{2m\pi x}{a} \sinh (D_m - \gamma_m y). \tag{10}$$

In Region I, the field must have the same $\mathbf{x}$ variation as in Region II, and must decay exponentially in $\mathbf{y}$ because a surface wave contains its power in a region around the dielectric slab. Then the scalar function to use with (3) is

$$f = A_m \cos \frac{2m\pi x}{a} \exp (-\alpha_m y), \quad m \geq 1 \tag{11}$$

$$k^2 = \beta^2 + (2m\pi/a)^2 - \alpha_m{}^2. \tag{12}$$

The field is readily written from (3) and the above scalar, plus $E_y = (\alpha\beta/j\omega\epsilon)$ exp $(-\alpha y)$, $E_z = (-\alpha^2/j\omega\epsilon)$ exp $(-\alpha y)$, $H_x = -\alpha$ exp $(-\alpha y)$. At the dielectric-air boundary $(y = b + c)$ this field must be continuous with the corresponding field in Region II, for all $\mathbf{x}$ and $\mathbf{z}$. Orthogonality in $\mathbf{x}$ allows individual terms to be equated. In this problem it is possible to match all the tangential components only if the fields are transverse with respect to $\mathbf{y}$, although it is natural to expect the higher modes to be derived from a $\mathbf{z}$-directed Hertzian vector. The $\mathbf{y}$ direction represents a virtual direction of propagation. Later it will be seen that the waveguide region needs a TM field to match the dominant mode E_y in Region II; to match the guide field, a TM field is needed here. At $y = b + c$, matching gives two equations which are solved to yield

$$\gamma_m \coth [D_m - \gamma_m(b + c)] = \epsilon_1 \alpha_m \tag{13}$$

so for $m \geq 1$ the condition $D_m > \gamma_m(b + c)$ must be valid. For $m = 0$, the equation reduces to the analogous form of the equation for the single slab

$$\gamma \tan [\gamma(b + c) - D] = \epsilon_1 \alpha. \tag{14}$$

If D were known, then this equation would allow γ to be calculated.

For Region III, the parallel plate waveguide region, the field is obtained from

$$f = C_n \sin \frac{n\pi x}{a} \cosh \delta_n y \tag{15}$$

$$\epsilon_1 k^2 = \beta^2 + (n\pi/a)^2 - \delta_n{}^2 \tag{16}$$

which gives a field, TM with respect to the $\mathbf{y}$ direction, and represents waves originating at the discontinuity (cell mouth) and traveling in the $\mathbf{y}$ direction. Since $\sqrt{\epsilon_1} ka < \pi$, *all modes* are evanescent. Again the "propagation" factor $\exp(-j\beta z)$ has been deleted for brevity.

The Approximate Physical Situation

To make the problem amenable to attack, the septa are assumed to be vanishingly thin. This assumption is reasonable physically as it is desirable to construct thin septa to avoid disturbing the TE surface wave. A second assumption neglects the reflected evanescent modes in the waveguide region. That is, the wave is attenuated so greatly in traveling from the guide mouth down to the guide end (ground plane) and back that the portion coming back can be discarded. From the equations to be given shortly it may be ascertained that the nth wave is attenuated by

$$\exp(-n\pi b/a)$$

in traversing the guide. For all practical designs, $b>a$ so even for $n=1$ the reflected wave is negligible.

The field will thus be approximated; the important components are

$$E_y = \sum_{n=1}^{\infty} \frac{C_n}{jw\epsilon\epsilon_1}\left[\left(\frac{n\pi}{a}\right)^2 + \beta^2\right] \sin\frac{n\pi x}{a} \exp(\delta_n y) \tag{17}$$

$$E_z = \sum_n -(\beta\delta_n C_n/w\epsilon\epsilon_1)\sin\frac{n\pi x}{a}\exp(\delta_n y) \tag{18}$$

$$H_x = \sum_n j\beta C_n \sin\frac{n\pi x}{a}\exp(\delta_n y) \tag{19}$$

Next this waveguide field is matched to the field in Region II, at the mouth of one parallel plate "cell," $y=b$. Only E_y, E_z, and H_x need be written.

$$\sum_{n=1}^{\infty}\frac{C_n}{jw\epsilon\epsilon_1}(\epsilon_1 k^2+\delta_n^2)\sin\frac{n\pi x}{a}e^{\delta_n b} = \sum_{m=0}^{\infty}\frac{B_m}{jw\epsilon\epsilon_1}(\epsilon_1 k^2+\gamma_m^2)\cos\frac{2m\pi x}{a}\sinh\lfloor D_m-\gamma_m b\rfloor \tag{20}$$

$$\sum_n \frac{-\beta\delta_n C_n}{w\epsilon\epsilon_1}\sin\frac{n\pi x}{a}e^{\delta_n b} = \sum_m\frac{\beta\gamma_m B_m}{w\epsilon\epsilon_1}\cos\frac{2m\pi x}{a}\cosh[D_m-\gamma_m b\rfloor \tag{21}$$

$$\sum_n j\beta C_n\sin\frac{n\pi x}{a}e^{\delta_n b} = \sum_m j\beta B_m\cos\frac{2m\pi x}{a}\sinh[D_m-\gamma_m b\rfloor. \tag{22}$$

These equations are the usual infinite simultaneous equations obtained in boundary value problems. Next multiply (22) by $k^2/\beta\omega\epsilon$ and add it to (20) to give a new equation which, when simplified, is

$$\sum_n \delta_n^2 C_n\sin\frac{n\pi x}{a}e^{\delta_n b} = \sum_m \gamma_m^2 B_m\cos\frac{2m\pi x}{a}\sinh[D_m-\gamma_m b\rfloor. \tag{23}$$

This equation and (21) will be used to obtain a solution. The right-hand side of each may be reduced to a single term due to the cosine orthogonality by multiplying each equation by $\cos 2q\pi x/a$ and integrating from **0** to **a**. Since the resulting series in n is not uniformly convergent, one may question the validity of term by term integration. The result is, however, correct as may be shown by a Green's theorem argument.[6] The n series is not orthogonal to $\cos 2q\pi x/a$, and an integral formula (Dwight 465) must be used. So (23) and (21) become

$$(\pi\gamma_q^2 B_q/2\epsilon_q)\sinh(D_q-\gamma_q b) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\frac{n\delta_n^2 C_n\exp(\delta_n b)}{n^2-4q^2} \tag{24}$$

$$(\pi\gamma_q B_q/2\epsilon_q)\cosh(D_q-\gamma_q b) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\frac{-n\delta_n C_n\exp(\delta_n b)}{n^2-4q^2} \tag{25}$$

with ϵ_q the Neumann number,[7]

$$\epsilon_q = \begin{cases}2 & q>0\\ 1 & q=0.\end{cases}$$

Note that for n even, $C_n\equiv 0$. The pair of equations is not yet in the proper form; two simple manipulations are needed.

$$\text{Insert } n^2-4q^2 = \left(\frac{a}{\pi}\right)^2(\delta_n^2-\gamma_q^2)$$

and multiply the second equation by γ_q, and add and subtract to the first equation of the pair with the result

$$(\gamma_q^2 a^2 B_q/2\pi\epsilon_q)\exp(D_q-\gamma_q b) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\frac{n\delta_n C_n\exp(\delta_n b)}{\delta_n+\gamma_q}\qquad q=0,1,2,\cdots \tag{26}$$

$$(\gamma_q^2 a^2 B_q/2\pi\epsilon_q)\exp(-D_q+\gamma_q b) = \sum_n\frac{-n\delta_n C_n\exp(\delta_n b)}{\delta_n-\gamma_q}. \tag{27}$$

For $q>0$ the $\exp(-D_q+\gamma_q b)$ represents an attenuated wave, a higher order evanescent wave which is reflected from the virtual ground plane at $y=b$. The amplitudes of these higher order reflected waves are small, and they will be neglected. Thus (27) is approximated by

$$(1/2\pi)\delta_{q0}\gamma_0^2 a^2 B_0\exp(-D_0+\gamma_0 b) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\frac{-n\delta_n C_n\exp(\delta_n b)}{\delta_n-\gamma_q}\qquad q=0,1,2,\cdots. \tag{28}$$

The two infinite sets of simultaneous equations, (26) and (28), are a good physical approximation to the problem and need to be solved for the coefficients B_q and C_n and wave numbers δ_1 and γ_0. These equations will be solved exactly by function-theoretic methods.

The Contour Integral Solution

A contour integral is written whose residues form the terms of the series in (28). Then if the integral can be uniquely and explicitly determined, the solution is immediate. This method has been used with appreciable

[6] E. A. N. Whitehead, "Theory of Parallel Plate Media for Microwave Lenses," *Proc. I.E.E.*, vol. 98, pt. III, pp. 133–140; 1951.

[7] G. N. Watson, "Bessel Functions," Cambridge University Press, Cambridge, Eng., p. 22; 1952.

success by Hurd[8,9] Whitehead,[6] and others.[10–12] It is especially valuable for problems involving a semi-infinite region and for these cases can include one reflected wave on each side of the boundary. A general discussion of the function-theoretic method is given by Karp.[13]

Consider a complex function $f(w)$ which

1) has simple poles at δ_n, $n=1, 3, 5, \cdots$
2) has simple zeros at γ_q, $q=1, 2, 3, \cdots$
3) is elsewhere absolutely convergent in the strict sense
4) tends uniformly to zero as $|w|\to\infty$
5) obeys edge conditions.

Then by choosing a set of contours of increasingly large radius, selected to avoid the poles,[10]

$$\int_C \frac{f(w)dw}{w-\gamma_q} = \delta_{q0}f(\gamma_0) + \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{R(\delta_n)}{\delta_n - \gamma_q} = 0 \quad (29)$$

where $R(\delta_n)$ is the residue of $f(w)$ at $w=\delta_n$.

If the function satisfies the above conditions, it is unique; hence (28) can be compared directly with (29), and it is found that

$$(1/2\pi)\gamma_0{}^2a^2B_0 \exp(-D_0+\gamma_0 b) = f(\gamma_0) \quad (30)$$

$$n\delta_n C_n \exp(\delta_n b) = R(\delta_n). \quad (31)$$

Similarly

$$\int_C \frac{f(w)dw}{w+\gamma_q} = f(-\gamma_q) + \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{R(\delta_n)}{\delta_n + \gamma_q} = 0 \quad (32)$$

which when compared with the set (26) gives the same formula for C_n and in addition

$$(-\gamma_q{}^2a^2B_q/2\pi\epsilon_q) \exp(D_q - \gamma_q b) = f(-\gamma_q). \quad (33)$$

Now by comparing (30) and (33) for $q=0$, a key equation is obtained:

$$-f(\gamma_0) = \exp(-2D_0 + 2\gamma_0 b) f(-\gamma_0). \quad (34)$$

This determinantal equation relates γ_0 and D_0.

To summarize, if the correct $f(w)$ were known, the mode amplitudes would be given directly by (31) and (33) and the determinantal equation would give γ_0 in terms of D_0. The next task is to construct the correct function. To this end two infinite products which display the needed zeros are defined.

$$\Pi_1(w) = \prod_{p=1}^{\infty} (w-\gamma_p)(-a/2p\pi) \exp(aw/2p\pi) \quad (35)$$

$$\Pi_2(w) = \prod_{\substack{p=1\\ \text{odd}}}^{\infty} (w-\delta_p)(-a/p\pi) \exp(aw/p\pi). \quad (36)$$

It will appear below that the exponential factors make the products strictly convergent. Then if $g(w)$ is an entire function with $g(\gamma_0)=1$,

$$f(w) = g(w)\frac{B_0\gamma_0{}^2a^2 \exp(-D_0+\gamma_0 b)\Pi_1(w)\Pi_2(\gamma_0)}{2\pi\Pi_1(\gamma_0)\Pi_2(w)} \quad (37)$$

which contains the proper zeros and poles and also satisfies (30). The investigation of the asymptotic behavior of $f(w)$ is given in reference 5 and the $g(w)$ is determined from the edge conditions[14] in the same place. For $p>1$

$$\delta_p \simeq p\pi/a, \qquad \gamma_p \simeq 2p\pi/a$$

so that

$$\Pi_1(w) \simeq \prod_{p=1}^{\infty} [1 - aw/2p\pi] \exp(aw/2p\pi) \quad (38)$$

which is absolutely convergent for all w by Example 1 in Whittaker and Watson.[15] Similarly for Γ_2. From Hansen,[5]

$$g(w) = \exp[(w-\gamma_0)a \ln 2/\pi]. \quad (39)$$

The determinantal (34) can now be solved for D in terms of γ_0. Inserting values for $f(\gamma_0)$ and $f(-\gamma_0)$ and solving for D gives the form

$$D = \gamma(b - a\ln 2/\pi) + (1/2j)\ln\frac{\Pi_1(-j\gamma)\Pi_2(j\gamma)}{\Pi_1(j\gamma)\Pi_2(-j\gamma)} \quad (40)$$

where the original γ is now used.

Note that no approximations were made here. Simplifying the last term yields the relationship between D and

$$D = \gamma(b - a\ln 2/\pi) + \sum_{p=1}^{\infty}(-1)^p\left[\sin^{-1}\frac{a\gamma}{p\pi} - \frac{a\gamma}{p\pi}\right]. \quad (41)$$

For most cases, $\gamma a/\pi \ll 1$. Then

$$D \simeq \gamma(b - a\ln 2/\pi) - (1/6)(\gamma a/\pi)^3. \quad (42)$$

This expression is the desired relationship among D, γ, and a. Note that b is not involved due to the neglect

[8] R. A. Hurd, "Propagation of an electromagnetic wave along an infinite corrugated surface," *Can. J. Phys.*, vol. 32, pp. 727–734; December, 1954.

[9] R. A. Hurd and H. Gruenberg, "H-plane bifurcation of rectangular waveguide," *Can. J. Phys.*, vol. 32, pp. 694–701; November, 1954.

[10] Z. Szekely, "Junction of Two Rectangular Waveguides," M. S. Thesis, Dept of Elec. Eng., Univ. of Toronto; May, 1953.

[11] L. Brillouin, "Wave guides for slow waves," *J. Appl. Phys.*, vol. 19, pp. 1023–1041; November, 1948.

[12] F. Berz, "Reflection and refraction of microwaves at a set of parallel metallic plates," *Proc. I.E.E.*, vol. 98, pt. III, pp. 47–55; 1951.

[13] S. N. Karp, "An Application of Sturm-Liouville Theory to a Class of Two-Part Boundary Value Problems," Rep. BR-13, New York Univ., New York, N. Y., August, 1955.

[14] The field must be singular in the proper manner at the septa edges.

[15] E. T. Whittaker and G. N. Watson, "Modern Analysis," Cambridge University Press, Cambridge, Eng., p. 34, 1952.

of evanescent reflected waves in the waveguide region. Result (42) is inserted into (14) to give the final transcendental equation determining γ. This equation is

$$\gamma c \tan \left[\gamma(c + a \ln 2/\pi) + (1/6)(\gamma a/\pi)^3\right] = \epsilon_1 \alpha c. \quad (43)$$

The equation can be solved by a perturbation scheme, first finding γ without the cubic correction term, then using this value in the complete equation and calculating a more correct γ. It is found that the cubic term has negligible effect for all γa where the theory is valid. Thus an excellent approximation to (43) is

$$\gamma c \tan \gamma(c + a \ln 2/\pi) = \epsilon_1 \alpha c \quad (44)$$

and the "effective height" c^* is given very simply by

$$c^* = c + a \ln 2/\pi. \quad (45)$$

An example, which will be of use later, has been chosen. In this example $\epsilon_1 = 4$ and $kc = 0.379$ which would yield a β/k of 1.05 if the slab were placed directly on the ground plane. Fig. 4 contains a plot of β/k vs plate

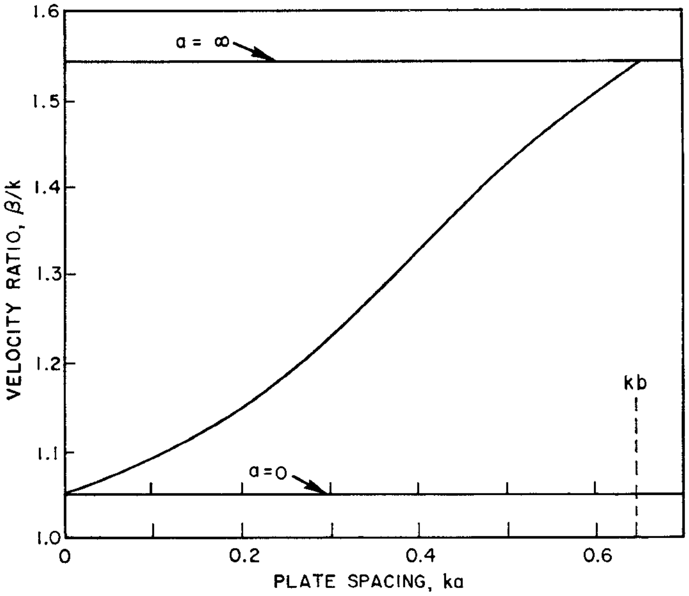

Fig. 4—Variation of propagation constant with septum spacing *ka*.

spacing for the example. When the plate spacing ***a*** becomes comparable with the plate height ***b***, the evanescent modes which were not accounted for in the theory become important. This fact is observed in the graph, as the true β/k can never exceed the value produced by the entire slab thickness, regardless of plate spacing. That the theory did not allow for reflected modes higher than the first in Region II (above the plates) is an important fact because in practice ***b*** will usually be greater than ***a***, but ***c*** will be less than ***a***.

The effective height c^* obtained is, except for the cubic correction term, just that obtained from the electrostatic problem of a semi-infinite capacitor. Fringing of the field between the inner and outer plates is equivalent[16] to adding a section of plate, with uniform field, of length ($a \ln 2/\pi$). The virtues of the function-theoretic approach are that it indicates exactly what approximations are used to obtain an answer and that the mode amplitudes can be calculated. Mode amplitudes have been calculated[5] for the example quoted; Table I compares the higher mode amplitudes to the

TABLE I

AMPLITUDE RATIOS AT INTERFACE

m	Ratio at Interface	Ratio at Cell Mouths
1	0.032	0.208
2	0.0015	0.0780
3	0.00010	0.0435
4	0.000009	0.0284

dominant mode amplitude at the dielectric-air interface, and at the cell mouths. The dominant mode in the dielectric of Region II originates at the interface and decays exponentially toward the waveguide cells. On the other hand, the higher order modes in the same region originate at the cell mouths, where they are needed to match the boundary conditions, and decay toward the interface. Hence Table I indicates that the *higher modes are insignificant at the interface but appreciable at the cell mouths.*

CONCLUSION

The Single Slab Arbitrary Polarization Structure is a practical structure which should find use in surface wave antennas and other devices. Finite spacing of the plates lowers the effective short circuit plane by an amount proportional to the plate spacing. This fact and the data presented allow a structure to be designed for a desired β/k ratio. The function-theoretic technique used also divulged the mode amplitudes in each region. In the parallel plate guide and just above the guides appreciable quantities of higher modes exist. At the trapping interface, higher order modes are negligible; thus this structure would not degrade the performance of an antenna.

A more precise theory will need to be cognizant of reflected evanescent waves in the dielectric region.

[16] W. R. Smythe, "Static and Dynamic Electricity," McGraw-Hill Book Co., Inc., New York, N. Y., p. 103, 1939. See prob. 26.